# Destabilization of Neutron Stars by Type I Dimension Bubbles

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## Abstract

An inhomogeneous compactification of a higher dimensional spacetime can result in the formation of type I dimension bubbles, i.e., nontopological solitons which tend to absorb and entrap massive particle modes. We consider possible consequences of a neutron star that harbors such a soliton. The astrophysical outcome depends upon the model parameters for the dimension bubble, with a special sensitivity to the bubble's energy scale. For relatively small energy scales, the bubble tends to rapidly consume the star without forming a black hole. For larger energy scales, the bubble grows to a critical mass, then forms a black hole within the star, which subsequently causes the remaining star to collapse. It is possible that the latter scenario is associated with core collapse explosions and gamma ray bursts.

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## I. INTRODUCTION

An inhomogeneous compactification of a higher dimensional spacetime with compact extra dimensions may result in the production of dimension bubbles[1]-[4] inhabiting the effective 4d spacetime. A dimension bubble may be either of the type I or type II variety, depending upon the parameters of the effective potential[4]. From a 4d perspective, a type I(II) dimension bubble is a nontopological soliton characterized by a scale factor for the extra dimensions which becomes much larger (smaller) inside the bubble than outside. For example, if the characteristic size of the extra dimensions varies from a Planck size to a $\mathrm{TeV}^{-1}$ size across the bubble wall, the scale factor can change by roughly 16 orders of magnitude with the size of the extra dimension remaining everywhere microscopic. Massive fermions and bosons in the effective 4d theory have masses varying inversely with the extra dimensional scale factor[4], so that they tend to become trapped inside a type I bubble, as in the case with the nontopological soliton of the type studied by Frieman, Gleiser, Gelmini, and Kolb[5]. Associated with the rapidly varying extra dimensional scale factor within a type I bubble wall is a very strong, short ranged attractive force. Therefore, massive particles that are incident upon a type I bubble tend to be absorbed by it. As energy is absorbed by the bubble, its mass and size both increase.

An interesting astrophysical situation can emerge if a type I dimension bubble is situated within an environment of high particle density, such as the core of a neutron star. As with the case of a Q-ball lodged within a neutron star[6, 7], the soliton may consume the star or form a black hole which then consumes the star. However, the solitonic properties, particle absorption rates, and energy absorption processes for dimension bubbles are quite different from those for Q-balls, and we focus attention here upon the consumption of a neutron star by a type I dimension bubble. We begin with an expression for the mass absorption rate $\dot{M}$ for a type I dimension bubble at the center of a neutron star, in terms of the Fermi momentum $p_F$ and the bubble radius $R$. The Fermi momentum can be related to the neutron number density $n$, so that we obtain a simple approximate proportionality between $\dot{M}$ and $R^2$. For a bubble that rapidly adjusts its radius so that it is approximately in mechanical equilibrium, the bubble mass $M$ is proportional to $R^2$, and $\dot{M}/M$ becomes a constant which depends upon the bubble wall tension $\sigma$. Different scenarios can ensue, depending upon the value of $\sigma$.

## II. SPHERICAL ACCRETION OF NEUTRON STAR BY DIMENSION BUBBLE

For convenience we take a simplistic view of a neutron star as a ball of neutrons with typical mass $M_{NS} \approx 1.4 M_{\odot} \approx 1.57 \times 10^{57} GeV$, a typical radius $R_{NS} \approx 10km \approx 5 \times 10^{19} GeV^{-1}$, with a neutron volume number density $n \approx \frac{M_{NS}/m}{(4/3)\pi R_{NS}^3} \approx \pi \times 10^{-3} GeV^3$, where $m$ is the neutron mass. (We use natural units where, approximately, 1fm $= 5GeV^{-1}$, $1GeV^{-1} = 6.6 \times 10^{-25}$ sec $= 2.1 \times 10^{-31}$yr.) A slow moving dimension bubble that gets gravitationally captured by a neutron star (NS) will be accelerated to a high speed (near the escape speed for the NS) at its surface. Associated with the high speed impact will be a large frictional force dissipating the bubble's energy, and we will assume that the bubble quickly finds itself at the center of the NS. The various simplifying assumptions are made to allow us to extract some of the essential features characterizing the interaction between the dimension bubble and the NS.

Now consider a type I dimension bubble of mass $M$ and radius $R$ sitting at the center of the NS, spherically accreting the star's mass. We consider the NS to be a degenerate Fermi gas with a volume number density of particles with energy between $E$ and $E + dE$ given by $\mathcal{N}(E)dE$. Within a time $dt$ a volume $dV$ of a spherical annulus of the NS falls into the bubble at a rate $\dot{V} = dV/dt$, carrying with it an amount of energy $dM$ which increases the bubble mass by an amount $dM$ at a rate $\dot{M} = dM/dt$. We can then write the rate of energy absorption by the bubble per unit of particle energy as

$$\frac{d\dot{M}}{dE} = \mathcal{N}(E) E \dot{V} \tag{1}$$

For the radial infall of neutrons we have $\dot{V} = 4\pi R^2 v(E)$ with particle velocity $v(E) = p/E = (E^2 - m^2)^{1/2}/E$, so that

$$\dot{V} = 4\pi R^2 \frac{(E^2 - m^2)^{1/2}}{E} \tag{2}$$

In the previous analysis one has to correct however by two effects: only half of the particles move in the direction of the bubble (this gives a factor of one half). Second, of those particles that move in the direction of the bubble, they do so at an angle. Given that the average of the cosinus of the angle between the normal and the velocity is also a half, eq.(1) is to be corrected by a factor of $\frac{1}{4}$. The number density $\mathcal{N}(E)$ is

$$\mathcal{N}(E) = 2 \cdot \frac{1}{2\pi^2} \frac{E(E^2 - m^2)^{1/2}}{e^{\beta(E - E_F)} + 1} \tag{3}$$

where the factor 2 represents the number of spin degrees of freedom per particle and $E_F$ is

the Fermi energy. We consider the low temperature $\beta \to \infty$ limit where

$$\frac{1}{e^{\beta(E-E_F)}+1} \approx \left\{ \begin{array}{l} 1,\ E < E_F \\ 0,\ E > E_F \end{array} \right\} \tag{4}$$

Integrating (1), corrected by the aboved mentioned factor of $\frac{1}{4}$, and using (2)-(4) we obtain

$$\dot{M} = \int_m^\infty \mathcal{N}(E) E \dot{V}\, dE = \frac{p_F^4 R^2}{4\pi} \tag{5}$$

where $p_F = (E_F^2 - m^2)^{1/2}$ is the Fermi momentum. The Fermi momentum is related to the volume number density $n$ of neutrons by $p_F = (3\pi^2 n)^{1/3}$. Taking $n \approx \pi \times 10^{-3} GeV^3$, we have $p_F \approx .45\ GeV$, $E_F \approx 1.04\ GeV \sim m$, and (5) gives

$$\dot{M} \approx\ 8n^{4/3} R^2 \approx (3.7 \times 10^{-3}\ GeV^4) R^2 \tag{6}$$

The result in (6) can be compared to a rough estimate obtained from the simple expression

$$\dot{M} \sim \frac{1}{4} \rho_{NS} \dot{V} \sim \pi R^2 \rho_{NS} v \tag{7}$$

where we take $\dot{V} \sim 4\pi R^2 v$ and again insert the above mentioned factor of $\frac{1}{4}$. Using $\rho_{NS} \sim nE_F \sim mn$ for the average energy density of particles in the neutron star and $v \sim p_F/E_F \sim .43$ for a typical velocity of a particle falling radially into the bubble, the simple expression in (7) gives $\dot{M} \sim (4 \times 10^{-3} GeV^4) R^2$, which is very close to the result in (6).

### A. Quantum Reflection

If the size of the extra dimension(s) is much larger inside the bubble than outside, the mass of the particle is greatly reduced inside the bubble. In effect, at the classical level, the particle experiences an enormous attractive force toward the bubble's interior within the bubble wall. Quantum mechanically, we can consider an associated attractive potential for the bubble's interior due to the decrease in particle mass. But even an attractive potential will generally give rise to some probability of reflection, with a corresponding reflection coefficient $\mathcal{R}$. Here, we wish to get a rough idea of how important this quantum reflection might be for particles being absorbed by a dimension bubble.

To do so, we ignore particle spin and polarization effects and consider, for simplicity, a one dimensional problem of relativistic spinless bosons obeying the Klein-Gordon equation $(\Box + m_{eff}^2(x))\phi = 0$ scattering from a flat bubble wall located at $x = 0$, which separates the bubble "exterior" for $x < 0$ and the bubble "interior" for $x > 0$. We approximate the position

dependence of the boson mass with a step function, $m_{eff}^2(x) = m^2[1-\theta(x)]$. In the "exterior" region ($x < 0$) we assume there to be a beam of particles of mass $m$ incident upon the wall from the left, propagating in the $+x$ direction, along with a reflected beam propagating in the $-x$ direction. For the "interior" region ($x > 0$) there is a beam of transmitted massless particles propagating in the $+x$ direction. We then have the plane wave solutions

$$\begin{aligned} \phi_{ext} = \phi_0 + \phi_1 &= A_0 e^{i(Et-px)} + A_1 e^{i(Et+px)}, \ (x<0) \\ \phi_{int} = \phi_2 \qquad &= A_2 e^{i(Et-p'x)}, \qquad\qquad\qquad (x>0) \end{aligned} \tag{8}$$

where $E^2 = p^2 + m^2 = p'^2$. Demanding continuity of $\phi$ and $\partial\phi/\partial x$ at $x = 0$ leads to

$$\frac{A_1}{A_0} = -\left(\frac{E-p}{E+p}\right), \qquad \frac{A_2}{A_0} = \frac{2p}{E+p} \tag{9}$$

We define the reflection coefficient $\mathcal{R} = j_1/j_0$ as the ratio of the reflected current density $j_1 = i\phi_1^* \overleftrightarrow{\partial_x} \phi_1$ to the incident current density $j_0 = i\phi_0^* \overleftrightarrow{\partial_x} \phi_0$, giving

$$\mathcal{R} = \left(\frac{E-p}{E+p}\right)^2 \tag{10}$$

Similarly, the transmission coefficient is found to be $\mathcal{T} = j_2/j_0 = 4Ep/(E+p)^2$ with $\mathcal{R} + \mathcal{T} = 1$. Taking typical values $p \sim p_F \sim m/2$, $E \sim E_F \sim 2p_F \sim m$, we obtain $\mathcal{R} \sim 1/9$, $\mathcal{T} \sim 8/9$. Although the reflection probability may be near 10 percent, we will neglect it in what follows.

## III. NEAR-EQUILIBRIUM BUBBLES

We again consider a type I dimension bubble where the size of the extra dimension(s) inside the bubble is much greater than that outside the bubble, i.e., $B_{in} \gg B_{out}$, where $B_{in(out)}$ denotes the scale factor of the extra dimension(s). Massive particle modes then have masses $m_{in} \ll m_{out}$, and we can focus on the effective radiation modes comprised of photons as well as particles with masses $m_{in} \ll |\vec{p}|$ that tend to stabilize the bubble against collapse. (We assume that the energy density of any nonrelativistic massive modes is negligible in comparison to the radiation energy density $\rho_{rad} \propto T^4$, where $T$ is the temperature inside the bubble.) We also assume that the energy density $\lambda$ associated with the value of the effective potential inside the bubble is negligible in comparison to the radiation energy density $\rho_{rad}$. The mass $M$ of such a bubble in mechanical equilibrium is given, approximately, in terms of the bubble radius $R$ by[3, 4]

$$M = 12\pi\sigma R^2 \tag{11}$$

where $\sigma$ is the surface energy density of the bubble wall. If we consider a bubble consuming matter that adjusts its radius sufficiently quickly to be considered in a state of near-equilibrium, we can use (11) so that (6) gives

$$\frac{\dot{M}}{M} \approx 0.2\frac{n^{4/3}}{\sigma} = \frac{K}{\sigma_{GeV}} \tag{12}$$

where $\sigma_{GeV}$ is a dimensionless number given in terms of $\sigma$ by $\sigma = \sigma_{GeV}(GeV^3)$ and

$$K = 0.2n^{4/3} \approx 1 \times 10^{-4} GeV \approx 1.5 \times 10^{20}\,\mathrm{sec}^{-1} \approx 5 \times 10^{27}\ \mathrm{yr}^{-1} \tag{13}$$

On dimensional grounds, we take a typical time scale associated with bubble equilibration to be $\tau_{eq} \sim \sigma^{-1/3}$, so that (12) is assumed to be roughly self consistent for $(\dot{M}/M) \lesssim \tau_{eq}^{-1}$, or $\sigma^{1/3} \gtrsim 10^{-1}\ GeV$. An integration of (12) indicates that the bubble acquires a mass $M$ after a time

$$t = \frac{\sigma_{GeV}}{K} \ln \frac{M}{M_0} \tag{14}$$

where $M_0$ is the initial bubble mass.

## IV. BLACK HOLE FORMATION, CORE COLLAPSE

We must consider the possibility that a growing dimension bubble will form a black hole before consuming the entire NS. For a spherical nonrotating bubble this occurs when the bubble mass reaches a value $M_{crit}$ such that $R = 2GM_{crit}$. Using this in conjunction with (11) gives

$$M_{crit} = \frac{M_P^4}{48\pi\sigma}, \qquad \frac{M_{crit}}{M_{NS}} \sim \frac{4.2 \times 10^{16}}{\sigma_{GeV}} \tag{15}$$

where $M_P = G^{-1/2} \sim 10^{19} GeV$ is the Planck mass. From (14) the time required for the bubble to evolve into a black hole is

$$t_{crit} = (\sigma_{GeV}/K)\ln(M_{crit}/M_0), \tag{16}$$

provided that $M_{crit} \lesssim M_{NS}$, or $\sigma^{1/3} \gtrsim 3.4 \times 10^5\ GeV$. For $\sigma^{1/3} \lesssim 3.4 \times 10^5\ GeV$, the bubble can devour the entire NS within a time

$$\tau \sim (\sigma_{GeV}/K)\ln(M_{NS}/M_0) \tag{17}$$

without first forming a black hole. (The logarithmic functions $\ln(M_{crit}/M_0)$ and $\ln(M_{NS}/M_0)$ are slowly varying functions of $M_0$, with $\ln(M_{NS}/M_0) \lesssim 130$ for $M_0 \gtrsim 1\ GeV$, and are therefore relatively insensitive to $M_0$.)

Table I presents some values (with $M_0 = 1GeV$) indicating $M_{crit}$, along with $\tau$ (amount of time required for bubble to consume entire NS without formation of a black hole) or $t_{crit}$ (amount of time required for bubble to grow within NS before formation of a black hole) for various values of the bubble energy scale $\sigma_{GeV}^{1/3}$.

| $\sigma_{GeV}^{1/3}$ | $\frac{M_{crit}}{M_{NS}}$ | $\tau$ (yr) | $\frac{t_{crit}(\text{yr})}{\ln(M_{crit}/M_0)}$ |
|---|---|---|---|
| $10^{-1}$ | $4.2\times10^{19}$ | $3\times10^{-29}$ | – |
| $10^{0}$ | $4.2\times10^{16}$ | $3\times10^{-26}$ | – |
| $10^{3}$ | $4.2\times10^{7}$ | $3\times10^{-17}$ | – |
| $10^{6}$ | $4.2\times10^{-2}$ | – | $2.2\times10^{-10}$ |
| $10^{9}$ | $4.2\times10^{-11}$ | – | $2.2\times10^{-1}$ |
| $10^{12}$ | $4.2\times10^{-20}$ | – | $2.2\times10^{8}$ |
| $10^{16}$ | $4.2\times10^{-32}$ | – | $2.2\times10^{20}$ |
| $10^{19}$ | $4.2\times10^{-41}$ | – | $2.2\times10^{29}$ |

TABLE I: Some characteristics associated with a type I dimension bubble inside a neutron star

If we use the Bondi-Hoyle accretion formula[8]

$$\dot{M} = 4\pi G^2 \rho_{NS} M^2 \tag{18}$$

to describe the spherical accretion of the NS remainder by the black hole, we find that the amount of time $t_{BH}$ required for the black hole mass to evolve from $M_{crit}$ to $M_{NS}$, i.e., the amount of time needed for the black hole to consume the entire NS, is

$$t_{BH} = (4\pi G^2 \rho_{NS})^{-1} \left( \frac{1}{M_{crit}} - \frac{1}{M_{NS}} \right) \tag{19}$$

which may vary from $t_{BH} \sim 10^{-3}$ sec for $\sigma_{GeV}^{1/3} = 10^6$ to $t_{BH} \sim 10^8$yr for $\sigma_{GeV}^{1/3} = 10^{12}$ to $t_{BH} \sim 10^{15}$yr for $\sigma_{GeV}^{1/3} = 10^{19}$.

It is possible that a neutron star will become destabilized by an evolving dimension bubble or black hole and a core collapse of the NS may lead to an observable photon burst from the collapsing/exploding star. However, the ultimate astrophysical outcome and phenomenology will be highly dependent upon the particle physics model parameters such as $\sigma$, as well as the astrophysics connected with the NS collapse and the potentially ensuing explosion.

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